7. Digital quantum circuits (pictorial)

7A. Basic elements

Quantum wire
Quantum wire bundle (n qubits)
Quantum wire bundle (alternate)

Classical wire
Entangled bit (ebit; Bell pair)

Quantum gate $\hat{U}$

Control gate U (control on $|1\rangle$)

Control gate U (control on $|0\rangle$)

Control-X (cNOT)

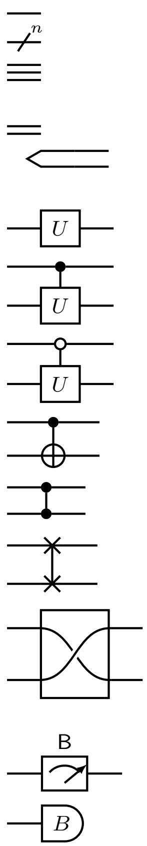
Control-Z (cZ)[1]

Swap gate

Swap gate (alternate)

Measurement in basis B

Measurement in basis B (alt)



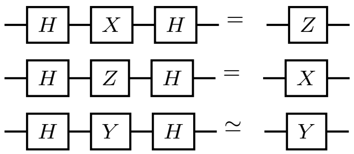
[1] controlled-Z operation is "symmetric" in the roles of control and target; hence the circuit representation by two dots.

7B. Circuit identities

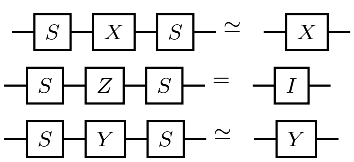
Proofs and checks: See (C69) and Mathematica/2021 RC Learn experiment/2022-11.1 circuit identities basic.nb

Basic and super useful

Pauli operator basis change



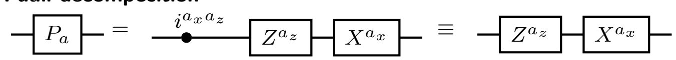
(−1 global phase)



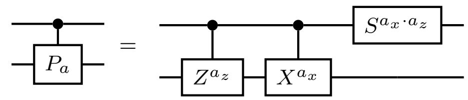
(i global phase)

(i global phase)

Pauli decomposition



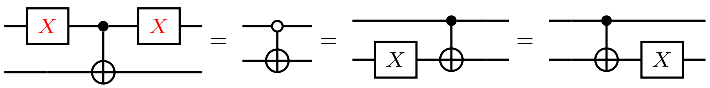
Pauli decomposition $P_a = i^{a_x a_z} X^{a_x} Z^{z_z}$ with $a = (a_x, a_z)$, see Sec. 3C. Note that for gates $P_a \cdot P_a^\dagger$, the $i^{a_x a_z}$ drops out. The global phase $i^{a_x a_z}$ for non control Pauli P_a can be ignored. It only applied to $a = (1\ 1)$ for $P_a = Y$ (C70-4)



Note that the phase kickback from the Y Pauli P_a shows up as a i phase on the control 1 state. Not this S gate only shows up for Y, ie $a = (1\ 1)$. (C70-4)

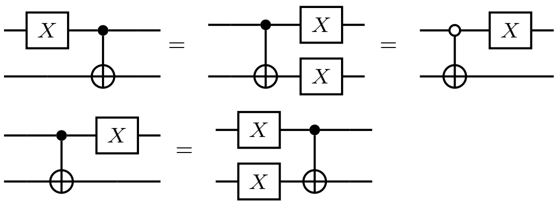
Controlled-NOT Gate (cNOT, cX)

cX + X

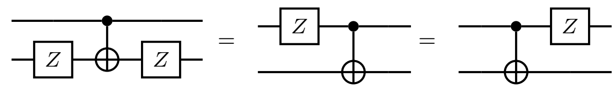


(C69-1)

"X travels forwards" from control to target (C69-2)

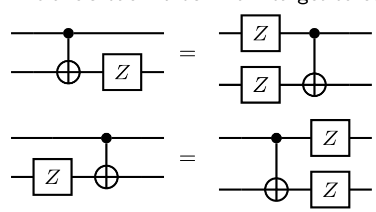


cX +Z

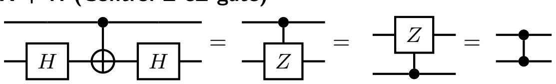


(C69-5)

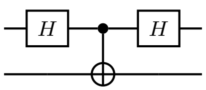
"Z travels backwards" from target to control (C69-5 / see above)



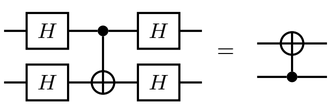
cX + H (Control-Z cZ gate)



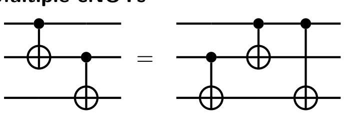
[10, Sec F2]



= TODO

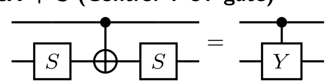


Multiple cNOTs



(C69-3) Note that $\mathrm{cX}_{12}\mathrm{cX}_{13} = |0\rangle\langle 0|_1 I + |1\rangle\langle 1|_1 X_2 X_3$. Think of having to push through the X on the control-2 only when control-1 is $|1\rangle\langle 1|$, then pushing X through cX_{23} yields X travels forwards and gives an X on each of the 2-3 wires.

cX + S (Control-Y cY gate)



Control-Y cY gateMaslov